CONSTANT TEMPERATURE FRONTS IN COMPOSITE BODIES

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The singularities of the advancement of isotherms in composite bodies are considered, and their application in thermophysical practice is shown.

Composite bodies are used extensively in engineering and in thermophysical practice. In the last case they are used to determine the heat conductivity λ by the method of regular mode bicalorimeters (plane, cylindrical, ball) and also in instruments for the dynamical determination of the thermophysical properties of bodies [1, 2]. Usually the λ built up on the shell case or the λ and a of the core are hence investigated.

The mentioned methods of determining λ and a are based theoretically on a whole series of assumptions, and factors distorting the result sometimes arise in their practical utilization, among which is, for example, the looseness of the shell fit on the core. This forces the researcher to introduce intermediate media (grease) and to take account of their effect.

This paper is aimed at establishing the regularity of constant temperature-front advancement Θ = idem in composite bodies and at showing the possibility of their utilization.

To do this, let us first turn to an analysis of the Lykov solution [3] about the symmetry temperature field of a three-composite plate (a plane bicalorimeter), which is described by the dependence

$$\Theta = \frac{T(x,\ \tau) - T_c}{T_0 - T_c} = \frac{2}{\mu_1 \psi_1} \cos\left(\mu_1 K_a^{-1/2} \frac{x}{l_2}\right) \exp\left(-\mu_1^2 \frac{l_1}{l_2} K_a^{-1} \frac{a_1 \tau}{l_1^2}\right) \quad (1)$$

for a core (enclosed body) under boundary conditions of the I or III kind in the regular stage.

Taking the logarithm and differentiating (1) with respect to τ for Θ = idem in sequence yields a formula to compute the displacement velocity of the constant temperature front [4],

$$v_\Theta = \left[\frac{\partial (l_1 - x)}{\partial \tau}\right]_\Theta = \frac{\mu_1 a_1}{l_2} K_a^{-1/2} \operatorname{ctg}\left(\mu_1 K_a^{-1/2} \frac{x}{l_2}\right). \quad (2)$$

Analysis of (2) shows that the displacement velocity of the isotherm Θ = idem in composite bodies depends on both their geometry (l_1, l_2) and on the thermophysical properties of the core and shell in the general case. For fixed body geometries and properties the velocity v_Θ in the steady thermal kinetics stage is determined just by the coordinate x of the point under consideration and by the boundary conditions.

If a characteristic equation for $0 \leq \mu_1 \leq \pi/2$, in the form

$$K_a^{1/2} \frac{c_1 \rho_1}{c_2 \rho_2} \cdot \frac{\mu_1}{\mathrm{Bi}} \left(1 + \frac{l_1}{l_2}\right) \operatorname{tg}\left(\mu_1 \frac{l_1}{l_2} K_a^{-1/2}\right) = 1 - \frac{\mu_1}{\mathrm{Bi}} \left(1 + \frac{l_1}{l_2}\right) \operatorname{tg}\mu_1 - K_a^{1/2} \frac{c_1 \rho_1}{c_2 \rho_2} \operatorname{tg}\mu_1 \operatorname{tg}\left(\mu_1 \frac{l_1}{l_2} K_a^{-1/2}\right), \quad (3)$$

is added to the dependence (2), then the combined solution of the system (2)-(3) for known l_1, l_2, a_1, c_1, c_2, ρ_1, ρ_2, Bi, and the calculated v_Θ permits determination of the temperature conductivity of the shell a_2.

Let us note that the velocity v_Θ is calculated by differentiation of the behavior of the experimental curve T = idem at the point x of the core in the coordinates $(l_1 - x) - \tau$, and the fact of regularization of

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the thermal mode is established by the equidistance of the path of the line sections for different T = idem (Θ = idem) in the coordinates mentioned [4].

It is easy to show that the system of equations needed to compute a_2 in the case of a cylindrical bicalorimeter by acting analogously to the above and by using the solution in [3], is

$$v_\Theta = \left[\frac{\partial (R_1 - r)}{\partial \tau}\right]_\Theta = \frac{\mu_1 a_1}{R_1} \frac{J_0\left(\mu_1 \frac{r}{R_1}\right)}{J_1\left(\mu_1 \frac{r}{R_1}\right)}, \tag{4}$$

$$J_0(\mu_1)\left\{\mathrm{Bi}\cos\left[K_a^{1/2}\left(\frac{R_2}{R_1}-1\right)\mu_1\right] - K_a^{1/2}\frac{R_2}{R_1}\mu_1 \sin\left[K_a^{1/2}\left(\frac{R_2}{R_1}-1\right)\mu_1\right]\right\} -$$

$$- K_a^{1/2}\frac{c_1\rho_1}{c_2\rho_2} J_1(\mu_1)\left\{\mathrm{Bi}\sin\left[K_a^{1/2}\left(\frac{R_2}{R_1}-1\right)\mu_1\right] + K_a^{1/2}\frac{R_2}{R_1}\mu_1\cos\left[K_a^{1/2}\left(\frac{R_2}{R_1}-1\right)\mu_1\right]\right\} = 0, \tag{5}$$

$$0 \leqslant \mu_1 \leqslant 2.4048.$$

For a ball bicalorimeter, the known solutions [3] are presented just for Bi = ∞. Then considering $K_a^{1/2}[(R_2/R_1) - 1]$ irrational, we have

$$v_\Theta = \left[\frac{\partial (R_1 - r)}{\partial \tau}\right]_\Theta = \frac{\mu_1^2 a_1}{\left(\frac{R_1}{r} - \mu_1 \operatorname{ctg}\mu_1 \frac{r}{R_1}\right) R_1}, \tag{6}$$

$$K_a^{1/2}\mu_1 \operatorname{ctg}\left[K_a^{1/2}\left(\frac{R_2}{R_1}-1\right)\mu_1 + 1\right] + K_a^{1/2}\frac{c_1\rho_1}{c_2\rho_2}(\mu_1\operatorname{ctg}\mu_1 - 1) = 0, \tag{7}$$

$$0 \leqslant \mu_1 \leqslant \pi.$$

The selection of the quantity μ_1 in (2)-(7) is subject to the condition $v_\Theta < \infty$ for $x/l_1 > 0$ and $r/R_1 > 0$ [4].

As is seen from (2)-(7), the determination of the quantity a_2 is not associated with any constraints on the core — shell pair and has a rigorous foundation.

Now, let us turn to an analysis of the regularities of advancement of the fronts Θ = idem in the particular case when the core is surrounded by either a sufficiently thick shell with arbitrary properties, or by a shell whose temperature conductivity is known to be below the temperature conductivity of the core. For brevity, let us call such a nucleus heat-insulated.

Then if the term $\mu_1 K_a^{-1/2}(x/l_2)$ becomes a small quantity (it is sufficient that $\mu_1 K_a^{-1/2}(x/l_2) \leq 0.42$), then (2) for the core of a plane bicalorimeter can be reduced to

$$v_\Theta = \frac{\frac{\mu_1 a_1}{l_2}K_a^{-1/2}}{\operatorname{tg}\left(\mu_1 K_a^{-1/2}\frac{x}{l_2}\right)} \cong \frac{\frac{\mu_1 a_1}{l_2}K_a^{-1/2}}{\mu_1 K_a^{-1/2}\frac{x}{l_2}} = \frac{a_1}{x}. \tag{2'}$$

There is no μ_1 in (2') and this indicates that the velocity v_Θ is now independent of either the shell properties or the thermal circumstances at the bicalorimeter boundary, i.e., on the quantity Bi. The latter substantially simplifies an experiment to determine the temperature conductivity a_1 of the core, whose magnitude it is not difficult to establish on the basis of (2') for a velocity v_Θ first calculated at the point x on the T = idem isotherm in the regularized kinetics domain. It is also clear that the need to perfect the thermal contact between the core and the shell drops out here.

Starting from the fact that the Bessel function of the first kind for $\mu_1(r/R_1) \ll 1$ (the case of a heat-insulated cylindrical core) and $\nu \geq 0$ can be represented as

$$J_\nu\left(\mu_1\frac{r}{R_1}\right) \approx \frac{1}{\Gamma(\nu+1)}\left(\frac{\mu_1\frac{r}{R_1}}{2}\right)^\nu \quad (\Gamma \text{ — is the gamma function}),$$

we obtain the following relationship:

$$v_\Theta = 2\frac{a_1}{r}. \tag{4'}$$

In a heat-insulated ball core we have

$$v_\Theta = 3\,\frac{a_1}{r}\,. \tag{6'}$$

According to our proofs [4], formulas (2'), (4'), (6') describe the rate of displacement of the isotherm Θ = idem in the quasistationary mode for the corresponding noncomposite bodies. Hence, the boundary conditions of the I and III kind can be replaced in experimental practice by boundary conditions of the II kind by the method described here and by maintenance of the contant heat flux density q = const, which is not subject to measurement, in the outer surface of the heat-insulated experimental bodies.

In the regular thermal mode, when the time change in the temperature of the core is described by a simple exponential, taking account of the dependences (2'), (4'), (6') results in the deduction about the elliptical nature of the temperature distribution over the coordinate which holds for fixed τ in the core of a plane bicalorimeter of the form

$$\Theta_{\mathrm{pl}} = f_{\mathrm{pl}}(\mu_1)\sqrt{1-\mu_1^2 K_a^{-1}\left(\frac{x}{l_2}\right)^2},$$

and for cylindrical and ball cores, respectively,

$$\Theta_{\mathrm{cyl}} = f_{\mathrm{cyl}}(\mu_1)\sqrt{1-\frac{\mu_1^2}{2}\left(\frac{r}{R_1}\right)^2},$$

$$\Theta_{\mathrm{ball}} = f_{\mathrm{ball}}(\mu_1)\sqrt{1-\frac{\mu_1^2}{3}\left(\frac{r}{R_1}\right)^2}.$$

The latter should be taken into account in constructing the coordinate functions in approximate methods of solving the corresponding heat-conduction problems and computations, since the elliptic temperature distribution may include the whole core. This holds for $\mu_1 K_a^{-1/2}(l_1/l_2) \le 0.42$ in a plane bicalorimeter. Analogous estimates can be made for other bodies also.

NOTATION

Θ, $T(x, \tau)$, T_0, T_c	are the dimensionless and dimensional running temperature of the core, its initial temperature, and temperature of the surrounding medium;
l_1, l_2, R_1, R_2, $x(r)$	are the half-thickness of the plate-core and shell thickness, core radius and outer radius of a cylindrical (ball) bicalorimeter, running point of the core;
τ	is the time;
a_1, c_1, ρ_1, a_2, c_2, ρ_2	are the temperature conduction, specific heat, and density of the core and shell;
μ_1	is the first root of the characteristic equations;
v_Θ	is the velocity of the isotherm;
K_a	is the criterion of the inertial properties of the core and shell;
Bi	is the Biot number.

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